193: Estimating Average Sky View Factors of Urban Surfaces with Simple Geometric Parameters

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Abstract

The sky view factors (SVFs) of urban surfaces have been shown to associate with microclimatic conditions such as the level of daylighting and radiant cooling potential. Methods were found in previous researches to calculate the average sky view factor of the ground and building facades with simple geometrical measurement of the urban form. These methods are, however, only applicable to simple generic urban forms. This study therefore aims to derive a method to estimate the average SVF of the ground (SVF_g) and of building exterior walls (SVF_w) for more irregular urban forms with simple geometric measurements.

The study illustrates that the ratio of ground area to building exterior wall area (GA/WA) serves as a good indicator for SVFs. Results from regression analyses show that, for urban forms with homogeneous building heights, α[GA/(GA+WA)] allows reasonable estimations for the SVF_g, with α being approximately 1.342 for various built forms prototypes including courtyard, terrace, point tower and cruciform. With SVF_g being known, SVF_w can also be derived with an analytical equation proposed in this study.

Keywords: urban form, sky view factor, geometric parameters, digital elevation model

1. Research Objective

The implications of sky view factor (SVF) on the urban micro-climate have being repeatedly studied in the past decades (see, for example, Givoni 1998). A high SVF is generally regarded as beneficial to daylight access under the overcast sky since it increases exposure to diffuse solar radiation. It is also conducive to mitigating nocturnal urban heat island effect at night since it allows radiant cooling of urban surfaces under the dark sky at night (Oke 1987). In addition, the SVF of urban surfaces are also used to predict the diffuse and reflected components in solar radiation exchange in urban canyons (see, e.g. Masson 2000, Harman et al 2004).

Traditionally, fish-eye lens is used to capture a sky view in a real urban setting and SVF can then be measured from the image. With computer-aided design software, it is now possible to obtain fish-eyes images from computer models and therefore the SVFs of design schemes or imaginative urban forms can be measured. It is, however, time consuming to make such measurement for the whole urban area since the average of numerous points is required. Recently, Ratti (2001) developed an image processing technique to calculate the sky view of an urban area based on digital elevation models (DEMs), which expedites the process of calculating the average SVF of the ground (SVF_g).

Some methods have been developed in the past to calculate average sky view factor of urban surfaces with simple geometrical parameters. Sparrow and Cess (1970) showed that, for an urban canyon with an infinite length, a width of *W* and a homogeneous height of *H*, the average SVF of the ground (SVF_g) and that of the wall (SVF_w) can be calculated by:

$$SVF_g = \left[1+(\frac{H}{W})^2\right]^{\frac{1}{2}} - \frac{H}{W} \qquad \text{...(1)}$$

$$SVF_w = \frac{1}{2}\left\{1+\frac{W}{H}-\left[1+\left(\frac{W}{H}\right)^2\right]^{\frac{1}{2}}\right\} \qquad \text{...(2)}$$

Kanda et al (2005) showed that the SVF_g of a uniform urban array with square building footprints and a homogeneous height can be estimated from the height, the width and the spacing of building blocks with high degree of accuracy with the following set of empirical equations:

$$SVF_g = V_{loc}V_{\mathrm{mod}} \qquad \text{...(3)}$$

where

$$V_{\mathrm{loc}} = \cos\{\tan^{-1}(2H/L)\}\langle 2 - 4/\pi \tan^{-1}[\cos\{\tan^{-1}(2H/L)\}]\rangle,$$

$$V_{\mathrm{mod}} = 0.1120\lambda_{\mathrm{P}} V_{\mathrm{loc}} - 0.4817\lambda_{\mathrm{P}} + 0.0246 V_{\mathrm{loc}} + 0.9570.$$

H is the height of the buildings
L is the spacing between buildings
λ_p is the site coverage of the building cluster

For the average SVF of all urban canyon surfaces (SVF_{canyon}) in a uniform urban array with square footprints and homogeneous building height, Kondo et al (2001) showed with computer simulation that it is equal to:

$$SVF_{canyon} = GA/(GA+WA) \quad ...(4)$$

where GA and WA are ground area and building exterior wall area, respectively.

While the above equations provide convenient ways for calculating SVFs, they are only applicable to uniform urban arrays that are seldom, if at all, seen in real urban developments. This study therefore aims to formulate a method to estimate, with simple geometrical measurements, the SVF_g and SVF_w of more realistic urban forms.

2. Methodology

2.1 Choice of urban forms and geometrical measurements

As discussed in Section 1, the height (H) and width (W) of a street canyon, or correspondingly the building exterior wall area (WA) and ground area (GA) of a uniform urban array, are shown to have high relevance to the average sky view factor of the ground (SVF_g) in simple urban forms. SVF_g is expected to increase with GA since GA represents the size of the "canyon ceiling", and is expected to decrease with WA since WA represents obstruction to the sky. Therefore, the potential correlation between SVF_g and the ratio of GA to WA (GA/WA) are first explored.

Generic urban forms are used to investigate the potential relationship between SVF_g and GA/WA. As shown in Figure 1, courtyard, terrace, point tower, diamond and cruciform of equal height are used to represent different urban forms in this study. To introduce further variations, these urban forms are assigned with four benches of site coverage (0.11, 0.14, 0.25 and 0.5) and three benches of plot ratio (2, 4 and 6). As tabulated in Table 1, the above arrangement results in altogether 60 different urban forms.

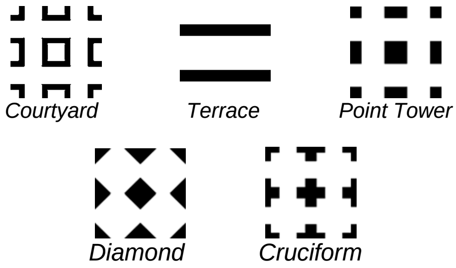

Fig 1. The five built form prototypes used in this study

Table 1: Urban forms analysed in this study (60 combinations)

Built form	Site Coverage	Plot Ratio
Courtyard	0.11	2
Terrace	0.14	4
Point Tower	0.25	6
Diamond	0.50 (0.33 for	
Cruciform	cruciform)	

2.2 Calculation of the average sky view factor of the ground

The average sky view factor of the ground is calculated by applying image-processing technique on digital elevation models (DEMs) as developed by Ratti (2001). DEMs of the 60 urban forms mentioned above are prepared and image-processed by Matlab software. Figure 2 shows an example of the processed image. The average sky view factor is obtained by analysing the average value of the grey tone of the image. Only the smallest repetitive unit at the centre will be used in order to minimise edge effect.

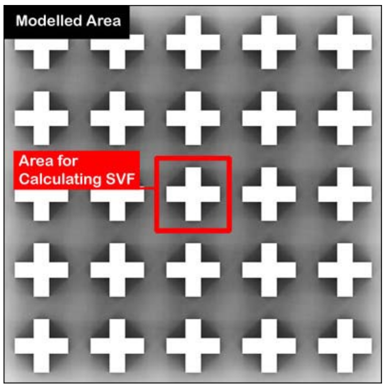

Figure 2: A processed digital elevation model with grey tone showing the sky view factor of the ground

2.3 Formulation of regression equations

The regression analysis begins with exploratory scatter plots and the study of the characteristics of extreme points. Figure 3 plots the average sky view factor of the ground against GA/WA for all urban forms.

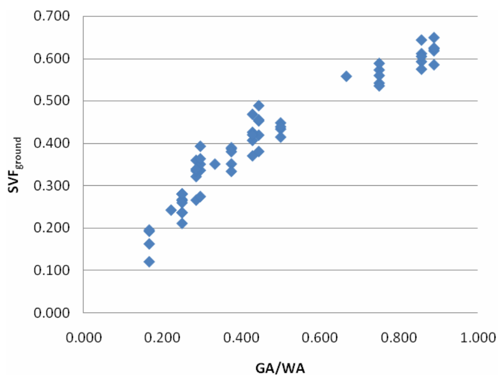

Fig 3. Scatter plot of the average sky view factor of the ground (SVF_g) against the ratio of ground area to building exterior wall area (GA/WA)

In addition to coherence with the shape of the plots, any equation that describes the relationship between SVF_g and GA/WA should fulfil the following conditions:

1. When GA/WA approaches 0, SVF_g should also approach 0; and
2. When GA/WA equals infinity, no building exists and therefore SVG_g should equal 1.

An inverse function with GA/WA being the independent variation is expected to fulfil these conditions. A quadratic function with arctan(GA/WA) as the independent variable and with no constant term is also expected to fulfil them. Regression analyses are therefore performed on SPSS based on these two functions.

2.4 Estimation of the average sky view factor of building exterior wall

Assuming that SVF_g can be found, the average sky view factor of building exterior wall (SVF_w) can be estimated based on an analytical equation. Taking a closer look to Equation (4) in Section 1, it actually provides a method to derive SVF_w from GA, WA and SVF_g for a uniform urban array with homogeneous height because:

$$SVF_{canyon} = GA/(GA+WA)$$

$$\frac{(SVF_g \times GA + SVF_w \times WA)}{(GA+WA)} = \frac{GA}{GA+WA}$$

$$(SVF_g \times GA + SVF_w \times WA) = GA$$

$$SVF_w = \frac{GA}{WA} \times (1 - SVF_g) \quad \text{… (5)}$$

As shown in Appendix 1, it can be analytically shown that this relationship is also applicable to any urban form with a homogeneous building height.

3. Findings

3.1 Estimation of the average sky view factor of the ground

Based on the scatter plot and the conditions above, exploratory regression analyses are performed. The following equations are found to satisfactorily represent the relationship between SVF_g and GA/WA:

$$SVF_g = \alpha\left(1 - \frac{1}{1 + GA/WA}\right) = \alpha\left(\frac{GA}{GA+WA}\right) \quad \text{… (6)}$$

$$SVF_g = \beta\left[\arctan^2\left(\frac{GA}{WA}\right) - \pi \arctan\left(\frac{GA}{WA}\right)\right] \quad \text{…(7)}$$

where α and β are the coefficients of the equations. Table 2 shows the coefficients of the best fitting curves and the corresponding R square values. Since Equation (6) has a higher R square value, smaller root-mean-square error (RMSE) and a simpler form, it will be adopted as the proposed model in this study and will be the sole focus in subsequent discussions.

Equation	Coefficient	Rsq. Value	RMSE
(6)	α = 1.342	0.942	0.033
(7)	β = -0.363	0.936	0.035

Table 2. Results of regression analyses on the best-fitting curves based on Equations (6) and (7)

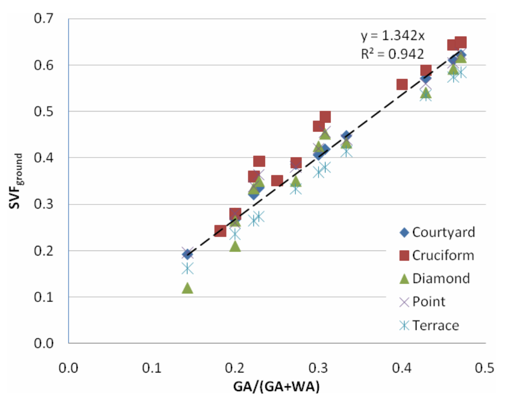

Fig 5. The average sky view factor of the ground (SVF_g) plotted against the ratio of ground area to ground-plus-building exterior wall area [GA/(GA+WA)] with the best-fitting curve of the proposed model [Equation (6)]

As illustrated in Figure 5, some built form prototypes have higher or lower SVF_g across the whole range of GA/(GA+WA). For example, the plots of cruciform are constantly higher than the average while terrace and diamond tend to be at the bottom. It is therefore possible to assign a different α value to each built form prototype to improve the accuracy of the prediction. Further regression analyses on each built form prototype suggests that the accuracy of the prediction increase considerably only for terrace and, to a lesser extent, cruciform, which is shown by the difference in root-mean-square errors (RMSE) on Table 3. Considering the relatively low RMSE in the original model and the small improvement by form-specific coefficients, it is opined that the original model would have been sufficient for providing reasonable estimates of SVF_g.

Built form	Form-specific α	RMSE	
		α =1.342	With form-specific α
Courtyard	1.353	0.013	0.013
Cruciform	1.437	0.044	0.032
Diamond	1.318	0.037	0.036
Point	1.362	0.026	0.025
Terrace	1.231	0.036	0.007

Table 3. Coefficient of best-fitting curve for each built form prototype and the corresponding roof-mean-square error

3.2 Estimation for the average sky view factor of building exterior wall

By substituting equation (6) to equation (5), SVF_w for urban forms with homogeneous height can be given by:

$$SVF_w = \frac{GA}{WA} \times \left(1 - \frac{\alpha \times GA}{GA + WA}\right) \quad \ldots (8)$$

To verify the predictive power of the equation, the values of SVF_w calculated with Equation (2) (Sparrow and Cess 1970) is plotted against the predicted value by Equation (8). As shown in Figure 6, Equation (8) provides reasonable prediction of SVF_s with a root mean square error of 0.021.

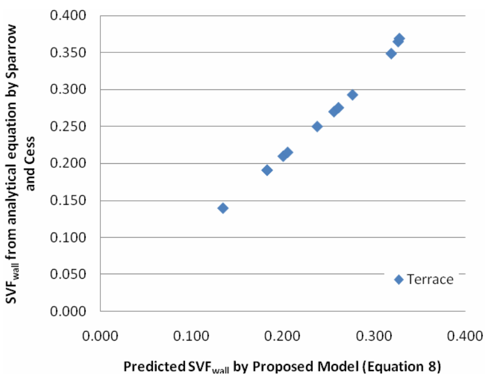

Fig 6. Values of SVF_w for terrace calculated by Equation (2) (Sparrow and Cess 1970) vs. that by Equation (8) (proposed model)

3.3 The effect of urban form irregularity

So far, the research assumed that building heights are homogeneous and buildings are uniformly distributed. As illustrated by Cheng et al (2006), both irregular building disposition (horizontal irregularity) and height variations (vertical irregularity) is expected to result in a higher SVF_g as compared to a uniform urban form with the same building shape, density and average height.

The proposed predictive equation uses GA and WA to estimate SVF_g, which obviously cannot capture the positive effect of neither horizontal nor vertical irregularities of buildings on SVF_g because GA and WA remain unchanged with these irregularities. Nevertheless, the proposed model can serve as an indicator for the minimum SVF_g that a certain irregular urban form can achieve.

To illustrate how much vertical irregularity increases SVF_g and therefore potentially weakens the predictive ability of the proposed equation, an urban form of PR 4 with height variation is analysed. As shown in Figure 7, "buildings" are assigned with two height benches, with the taller ones being two times the height of the short ones. Figure 8 plot the SVF_g of the same urban form with both homogeneous and variable heights. It shows that, for this particular case, increase in SVF_g due to height variable is small.

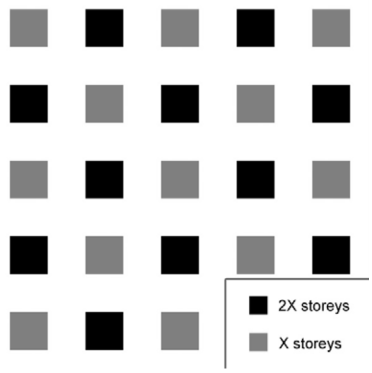

Fig 7. Height variation introduced for the simulation results in Figure 8

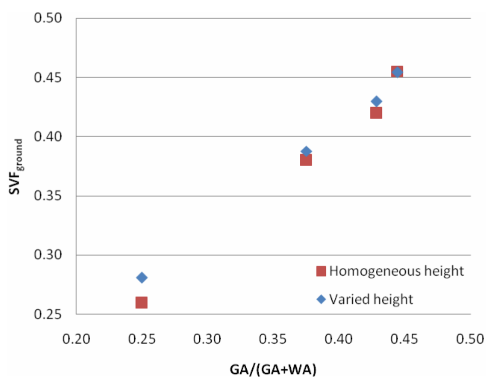

Fig 8. Effect of height variation on the average sky view factor of the ground

The effect of horizontal and vertical irregularity on SVF_w is less clear as previous research findings mentioned in this paper do not account for building height variations. Further research that measures SVF_w directly will be required.

4. Conclusion

This paper proposes predictive models for estimating the average sky view factor of the ground (SVF_g) and of building exterior walls (SVF_w) for urban forms will homogeneous building heights. Based on regression analyses on 60 urban forms covering a range of built forms, plot ratios and site coverages, it is showed that a predictive model with ground area (GA) and building exterior wall area (WA) as independent variables provides good estimate for SVF_g. This paper also provides an analytical equation for deriving SVF_w from SVF_g for urban forms with homogeneous building heights, which shows good agreements with findings from previous researches.

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APPENDIX 1 Relationship between the sky view factor of building exterior walls (SVF_w) and that of the ground (SVF_g) for urban forms with homogeneous building heights

Assuming that the ground with area *GA* and building façades with area *WA* are subdivided into infinitesimal parts g_i and w_j, respectively:

$$GA = \int_i \partial g_i$$

$$WA = \int_j \partial w_j$$

Let SVF_x be the average sky view factor of surface x and $VF_{y \to z}$ be the view factor of surface y to surface z,

$$\begin{aligned}
SVF_g &= \frac{1}{GA}\int_i SVF_{g_i}\partial g_i \\
&= \frac{1}{GA}\int_i (1 - VF_{g_i \to w})\partial g_i \\
&= \frac{1}{GA}\int_i \partial g_i - \frac{1}{GA}\int_i VF_{g_i \to w}\partial g_i \\
&= 1 - \frac{1}{GA}\int_i\int_j VF_{g_i \to w_j}\partial w_j \partial g_i
\end{aligned}$$

(by summation rule)

$$= 1 - \frac{1}{GA}\int_j\int_i VF_{w_j \to g_i}\partial g_i \partial w_j$$

(by reciprocality)

$$= 1 - \frac{1}{GA}\int_j VF_{w_j \to g}\partial w_j$$

(by summation rule)

$$\begin{aligned}
&= 1 - \frac{WA}{GA} \times VF_{w \to g} \\
&= 1 - \frac{WA}{GA} \times SVF_w
\end{aligned}$$

(since the "sky ceiling" is symmetrical to the ground area for urban forms with homogeneous height)

$$\therefore \quad SVF_w = \frac{GA}{WA} \times (1 - SVF_g)$$