109: Controlling the temperature structure in a naturally ventilated room with an underfloor air supply system

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Abstract
We investigate air flows in a room into which cool air is supplied at low momentum though inlets at a low level, and from which old air is extracted through openings at a high-level. The room contains a localised heat source and a distributed heat source. This situation may be analogous to an open-plan office equipped with an underfloor air supply system, which contains a cluster of printers/machines and office workers distributed across the floor, or an auditorium with a similar ventilation system which contains a distributed audience and a localised group of actors and lighting on a stage, for example. Using a combination of laboratory experiments and a theoretical model, we show that, in such conditions, if the localised heating is sufficiently strong compared to the distributed heating, the room will become stratified into two layers at steady state. The top layer is warmer than the lower layer which attains a temperature above that of the supply air. This temperature structure depends primarily on the rate of air flow through the space and the ratio of the distributed heating to the total heat flux. For a room with fixed heating, increasing the air flow rate raises the interface between the upper and lower layers, while cooling both layers. However, for a room with a fixed air flow rate and a fixed total heat flux, increasing the magnitude of distributed heating raises the interface and warms the lower layer without affecting the temperature in the upper layer. These steady state temperature structures are stable. The paper discusses how these principles of flow apply to a naturally ventilated room, whose air supply and extraction are accomplished by thermal buoyancy produced by the internal heat sources. It also discusses how, in such conditions, appropriate inlet and extract sizes may be determined for different heating ratios and supply air temperatures, which will effect the appropriate air flow rates that give thermal comfort and sufficient ventilation in the lower occupied zone, while keeping any pocket of warm air well above it.

Keywords: underfloor air supply, temperature structure, ventilation,

1. Introduction
A system of ventilation which has become increasingly common in modern buildings is one that supplies cool air at a low level and removes old air from a high level. In a mechanically operated system, air can be supplied and extracted by fans; whereas in a naturally operated system, air can be driven by buoyancy forces created by internal sources of heating/cooling, such as occupants and machinery. Various scenarios of ventilation in a room containing a combination of sources of heating and cooling have been studied. Cooper and Linden [1] studied the ventilation of a room containing two localised heat sources which connected to an exterior through a low level vent and a high level vent. They showed that if the two sources were of different strengths, then the room would become stratified into three layers at steady state. The temperature in each of the two upper layers was controlled by the magnitude of the source associated with that layer, while the bottommost layer attained the temperature of the supply air. For a fixed flow rate, adjusting the strength of one source to approach the other's would cause the two upper layers to merge, but would not affect the temperature in the bottommost layer. Lin and Linden [2], and Liu and Linden [3] studied the ventilation of a room containing a localised heat source, into which a localised jet of cold air [2] and a series of localised jets of cold air of the same strength [3] were introduced, and from which air was extracted through the ceiling. They showed that these conditions led to the room becoming stratified into two layers at steady state, with the localised heat source forming the upper layer and the cold jet(s) forming the lower layer. The depth and temperature of the lower layer were influenced by the entrainment of warm air from the upper layer into the lower layer caused by the cold jet(s) impinging the interface between the two layers. A theoretical model providing a means of control was presented in each of those studies.

Another situation which is commonly encountered is that involving a combination of localised heating and distributed heating. Such a combination exists, for example, in an auditorium containing a distributed audience and a localised group of actors and lighting on a stage, or in an underfloor heated room containing a cluster of occupants or machinery. Chenvidyakarn and Woods [4] studied the impact of such a combination of heating on air flow in a mechanically ventilated room, whose air supply and extraction were accomplished by fans (through the system so-called underfloor air-conditioning). The present paper addresses a situation in which air is driven instead by thermal buoyancy created by the internal heat sources, as may be found in passive buildings. To explore the basic principles of such a flow situation, we examine a simple case of a room containing one localised heat source and one distributed heat source, into which cool air relative to the interior is drawn through openings at a low level at low

momentum, and from which old air is driven through openings at a high level. We tackle a key design and control question: How can the temperature structure inside the room be controlled so that thermal comfort and sufficient ventilation are achieved?
To answer this question, in Section 2 we study the picture of flows that develop in the room using laboratory experiments. Then, in Section 3 we model this picture of flows quantitatively. The model is also tested with the laboratory experiments. In Section 4, we demonstrate how the model may be used to help develop a strategy for controlling the internal temperature structure in different heating conditions. Finally, we present conclusions in Section 5.

2. Laboratory experiments

Laboratory experiments are carried out to observe the picture of flows in a room which contains a combination of distributed heating and localised heating, and which is equipped with an underfloor air supply system. The experiments use a small-scale model and water, instead of air, as the working fluid. Dynamic similarity between the experiments and full-scale buildings is achieved by obtaining comparable magnitudes of Re, Pe and Ra [5, 6]. In the experiments, the ventilated room is represented by a clear acrylic tank of internal dimensions 17.5 x 17.5 x 25 cm. The tank has inlets at the base and outlets at the top. The room model is submerged in a reservoir of size 1 x 5 x 0.7 m, which acts as the ambient environment. Distributed heating is provided by an electronically controlled areal heating element, and localised heating is provided by a plume of warm water supplied though a nozzle. The temperature of the plume is controlled by a bath circulator. The plume flow rate is controlled by a peristaltic pump running at a speed such that a turbulent plume is achieved while producing negligible momentum and negligible mass flux (about 1 – 2 % of the net volume flux through the room model). During the experiments, the flux of distributed heating or the flow rate is varied. Flow visualisation is accomplished by shadowgraph imagery. The temperature structure in the room is monitored using thermocouples.
The experiments reveal the picture of flows that develop as the net volume flux of air flow though the room or the heating distribution changes. In a room containing a localised heat source alone, the localised source produces a plume of warm air which rises to the top while entraining the surrounding fluid. If the net volume flux of air flow through the room, Q_V, is less than the volume flux of the plume at the level of the extracts, $Q_{P(H)}$, $Q_V < Q_{P(H)}$, then some warm air will be trapped and form a layer in the upper part of the room at steady state (Fig 1a). The lower part of the room is unheated, and so attains the temperature of the inflow air. Increasing the volume flux of air flow through the room, $Q_V \rightarrow Q_{P(H)}$, will remove more warm air. Consequently, the upper layer becomes depleted and the interface between the upper and lower layer rises (Fig 1b). When the volume flux of air flow through the room equals or exceeds the volume flux of the plume at the height of the extracts, $Q_V \geq Q_{P(H)}$, the warm layer will be removed completely and the interface reaches the level of the extracts. The whole room now attains the temperature of the inflow air (Fig 1c).

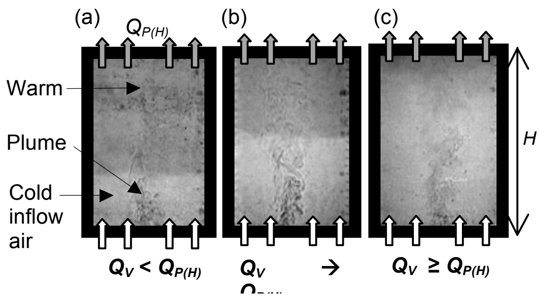


Fig 1. Impact of flow rate on interface height.

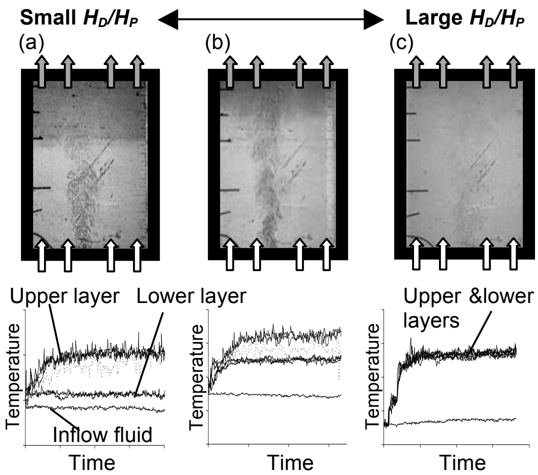


Fig 2. Impact of magnitude of distributed heating relative to localised heating.

However, if this room contains a combination of a distributed heat source of flux H_D and a localised heat source of flux H_P at the base, when the room is stratified, the lower layer will be heated by the distributed source and attain a temperature above that of the inflow air (Fig 2a). The top layer is now heated by the localised plume which has entrained heated air from the lower layer, and so attains a temperature above that of the lower layer. Since the top layer is heated by both distributed and localised sources while the lower layer is heated by the localised source alone, an increase in the strength of distributed heating compared to the localised heating will lead to the temperature in the lower layer increasing towards that of the upper layer (Fig 2b). Sufficiently large distributed heating will lead to the lower layer attaining the temperature of the upper layer, causing the room to become well-mixed (Fig 2c). The distributed source also produces convective currents which rise while mixing with air in the lower layer. If the localised heating is sufficiently strong compared to the distributed heating, these convective currents will be less buoyant than the upper layer. As a result, they cannot rise freely to the top, but instead will strike the interface between the upper and lower layers, penetrate somewhat into the upper layer, and thereby entrain some warm air from the upper layer into the lower layer. This penetrative entrainment leads to an increase in the depth of the lower layer. An increase in the magnitude of the distributed heating compared to the localised heating will increase the strength of this penetrative entrainment, and consequently deepen the lower layer and raise the interface (Fig 2b compared to 2a). Sufficiently strong distributed heating will cause the interface to rise to the level of the extracts, resulting in the room becoming well-mixed (Fig 2c).

These steady state temperature structures are stable, i.e. they may be upset by perturbations but will resume their stable forms once the perturbations are removed.

3. Quantitative model

3.1 Developing the model

We develop a model to describe the steady state flows in a stratified room and a well-mixed room as driven by thermal buoyancy created by the internal heat sources. To focus on flow principles, we model a lightweight building and neglect the effects of thermal mass and conduction/radiation heat transfer. Such additional effects may be incorporated into the model later according to specific cases. The heat fluxes modelled herein thus correspond to the convective component of the heat load.

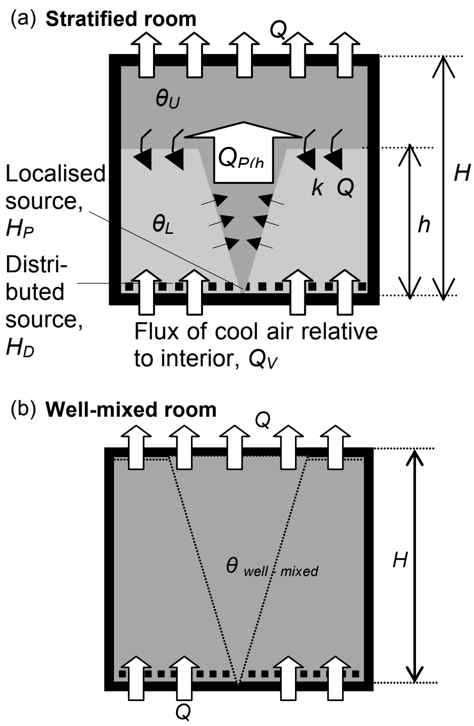


Fig 3. Schematics for quantitative model.

A schematic of the stratified room is shown in Figure 3a. The room contains a localised source of flux H_P (W) and a distributed source of flux H_D (W) at the base. In a naturally operated system, thermal buoyancy created by these heat sources draws cool air relative to the interior into the room at a volume flow rate Q_V (m^3/s), at low momentum though openings at a low level, while driving old warm air out of the room through openings at the top at the same rate. The vertical distance between the low-level inlets and high-level extracts is H (m). The lower part of the room attains a uniform temperature in excess of the inflow air θ_L (ºC). The localised plume produces a warm layer of a uniform temperature in excess of the inflow air θ_U (ºC) above a height h (m) from the base of the room. The volume flux supplied by the localised plume at this height h is $Q_{P(h)}$ (m^3/s). The penetrative entrainment at the interface carries warm air of volume flux Q_E (m^3/s) from the upper layer to the lower layer.

Mass conservation requires that, at steady state, the inflow volume flux equals the volume flux of the plume at the height of the interface less the volume flux carried from the upper to lower layer by the penetrative entrainment,

$$Q_V = Q_{P(h)} - Q_E \qquad (1)$$

If the room is naturally ventilated, the flow will be driven by thermal buoyancy associated with each of the two layers,

$$Q_V = A\sqrt{g\alpha[\theta_U(H-h)+\theta_L h]} \qquad (2),$$

where g (m/s^2) is gravitational acceleration, and α (1/K) is the volume expansion coefficient of air. The variable A is an effective opening area which takes into account pressure loss at the inlets and extracts. It is derived by tracing pressure along the streamline, which gives

$$A = \frac{\sqrt{2}c_s a_s c_e a_e}{\sqrt{(c_s a_s)^2 + (c_e a_e)^2}} \qquad (3),$$

where c_s and c_e are coefficients describing pressure loss at the inlets and extracts respectively, and a_s and a_e (m^2) are the areas of the inlets and extracts respectively.

The volume flux of the plume at the height of the interface is given by [7]

$$Q_{P(h)} = C\left(\frac{g\alpha H_P}{\rho C_p}\right)^{1/3} h^{5/3} \qquad (4)$$

where the dimensionless constant $C = 6/5\beta(9/10\ \beta)^{1/3}\pi^{2/3}$ describes the behaviour of the localised plume, with β being a dimensionless constant characterising the plume entrainment. The constant ρ (kg/m^3) is the density of air, and C_p (J/(kg °C)) is the specific heat capacity of air.

The volume flux carried across the interface by the penetrative entrainment increases with the strength of the distributed source but decreases with the temperature contrast between the upper and lower layers. This effect can be parameterised by [8]

$$Q_E = \frac{kH_D}{\rho C_p(\theta_U - \theta_L)} \quad (5),$$

where k is a dimensionless constant characterising the strength of penetrative entrainment.

The upper layer is heated by the plume which has entrained heated air from the lower layer. Thermal conservation in the upper layer gives

$$\begin{aligned} H_D + H_P &= \rho\, C_p Q_V \theta_U = \\ &\rho\, C_p Q_{P(h)}\theta_U - \rho\, C_p Q_E \theta_U \end{aligned} \quad (6).$$

The lower layer is heated by the distributed source. Thermal conservation in the lower layer gives

$$H_D = \rho\, C_p Q_{P(h)}\theta_L - \rho\, C_p Q_E \theta_U \quad (7).$$

We model a well-mixed room (Fig 3b), in which the interface reaches the height of the extracts, so that $h = H$; and in which the temperature in the lower layer reaches that of the upper layer, so that the whole room is heated by an equivalent of the total heat flux H_D+H_P. In this situation, the net volume flux and the interior temperature relative to that of the inflow air are given by

$$\begin{aligned} Q_V &= Q_{P(H)} = C\left(\frac{g\alpha H_P}{\rho C_p}\right)^{1/3} H^{5/3} \\ &= A\sqrt{g\alpha\theta_{well-mixed}H} \end{aligned} \quad (8)$$

and

$$\theta_{well-mixed} = \frac{H_D + H_P}{\rho C_p Q_V} \quad (9).$$

3.2 Testing the model

In testing the model, a series of experiments are carried out. The experimental setup has been described in Section 2. We test the impacts of different combinations of the flow rate and heating ratio on the interface height and interior temperature. In applying the plume model (Eq 4) to our experiments using a source of hot water to generate a plume, we have carried out a series of calibration experiments following the technique of Baines [9], and found that the plume has C = 0.23 and a virtual origin 0.016 m behind the actual source of hot water. The room model has a height H = 0.25 m. The following constants are taken [10]: gravitational acceleration g = 9.81 m/s^2, the volume expansion coefficient of water α = 0.138 x 10^{-3} 1/K, the density of water ρ = 999.1 kg/m^3, the specific heat capacity of water C_p = 4186 J/(kg ˚C), and penetrative entrainment constant k = 0.1.

To compare experiments with different boundary conditions, we use dimensionless variables. The temperature and volume flux in the stratified room are scaled relative to their values in the well-mixed room. This gives

$$Q'_V = h'^{5/3}\left(1 - \frac{kH'_D}{1 - H'_D}\right) \quad (10),$$

$$\theta'_L = \frac{H'_D(1+k)}{Q'_V} \quad (11)$$

and

$$\theta'_U = \frac{1}{Q'_V} \quad (12),$$

where the dimensionless variables are as follows: the dimensionless flow rate $Q'_V = Q_V / Q_{P(H)}$, the dimensionless temperature in the lower layer $\theta'_L = \theta_L/\theta_{well-mixed}$, the dimensionless temperature in the upper layer $\theta'_U = \theta_U/\theta_{well-mixed}$, the dimensionless flux of distributed heating $H'_D = H_D/(H_D+H_P)$, and the dimensionless interface height $h' = h/H$.

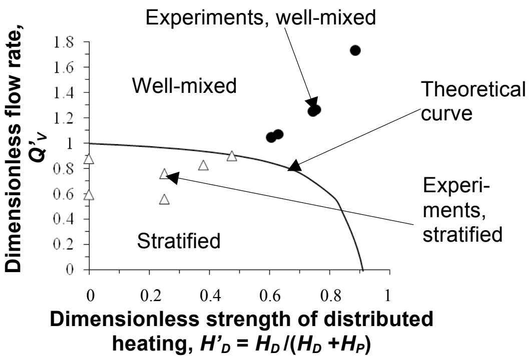
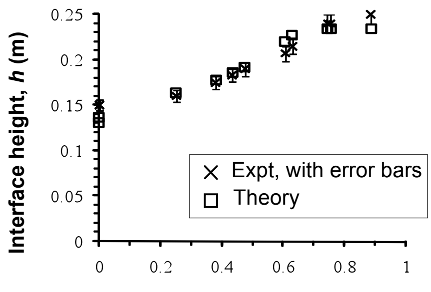


Fig 4. Testing boundary between well-mixed and stratified environment.

Figure 4 shows how, in the experiments, varying the flow rate and heating ratio changes a stratified room into a well-mixed room. (Each triangle represents a stratified room, and each circle represents a well-mixed room.) These experimental results are compared with a theoretical curve given by Eq 10, which predicts the transitional zone between the two temperature structures. The theoretical curve describes well the transition observed in the experiments.

Figure 5 compares the change in the interface height as observed in the experiments (crosses) with that predicted by the model (squares). Error bars are provided for the experimental data to represent the blurred region at the interface caused by the penetrative entrainment. The plot shows that the theoretical predictions agree well with the experimental observation; it may be argued that the discrepancies are due to the imprecision of the experimental data which is obtained through visual observation.



Dimensionless strength of distributed heating, $H'_D = H_D/(H_D + H_P)$

Fig 5. Testing change in interface height.

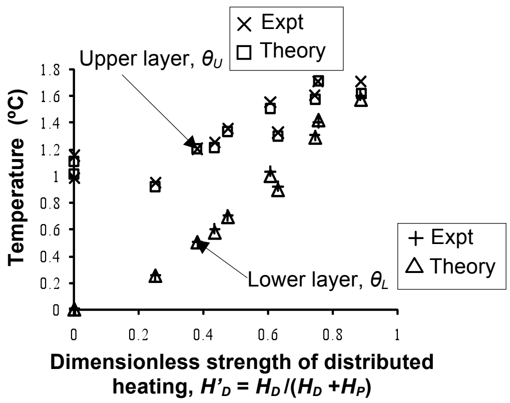


Fig 6. Testing change in interior temperature.

Figure 6 compares the change in the interior temperature as observed in the experiments (crosses) with that predicted by the model (solids). In the experiments, technical limitations dictate that the total heat flux has to be allowed to increase with the magnitude of distributed heating. This is because to keep the total heat flux constant while increasing the flux of distributed heating appreciably, the heat flux of the localised plume would have to be reduced so much that the resultant temperature difference between the upper and lower layers fell within the uncertainly range of the thermocouples. On the other hand, if the heat flux of the plume were to be kept sufficiently high throughout the experiments, the distributed heating would have to be impractically large to achieve the required heating ratios. Figure 6 shows the temperature in both upper and lower layers rising as the total heat flux increases with the strength of the distributed source. Taking into account the variation in the total heat flux, the theoretical predictions agree well with the experimental data. We demonstrate how the model may be applied to full-scale buildings in Section 4.

4. Applying the model

We have asked a key design and control question at the start of the article, namely: How can the internal temperature structure of a naturally ventilated room with an underfloor air supply system be controlled so that thermal comfort and sufficient ventilation are achieved? To illustrate how the model can help address this question and serve as a basis for developing effective control in full-scale buildings, let us consider this simple example. A lightweight auditorium has a volume of 3000 m^3. Its audience on the floor and on a balcony together provide distributed heating, while a group of actors and lighting on a stage together provide localised heating. Thermal buoyancy created by these heat sources combined draws fresh air at 17ºC into the auditorium, through inlets located on the floor underneath the seating. Old air is driven out by thermal buoyancy through openings located on the ceiling 10 m above the level of the inlets.

For demonstration purposes, the following design criteria are taken. To provide sufficient fresh air, the ventilation rate should be at least 0.83 m^3/s (i.e. one air change per hour). To achieve thermal comfort, the temperature in the occupied zone should be between 20 – 21ºC. Any pocket of uncomfortably warm air should be confined to at least 7 m above the floor level, to be well clear of the balcony audience. How should the ventilation system of the auditorium be controlled, if thermal comfort and sufficient ventilation is to be achieved when there are 8000 W of distributed heating from the audience and 7500 W of localised heating from the stage? How should the control be adjusted if the occupancy changes to 10000 W of distributed heating and 5500 W of localised heating?

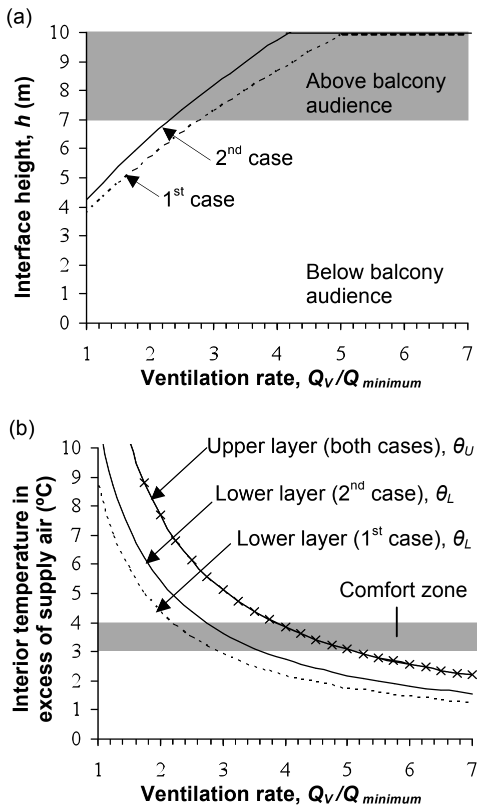


Fig 7. Example of model application.

The laboratory experiments and theoretical model have shown that, for a room with given heat fluxes and a given supply air temperature, its internal temperature structure can be controlled by adjusting the rate of air flow. In a naturally operated system, this can be achieved by adjusting the sizes of the inlets and extracts. We take the following values of constants [10]: gravitational acceleration g = 9.81 m/s^2, the volume expansion coefficient of air α = 3.5 x 10^{-3} 1/K, the density of air ρ = 1.2 kg/m^3, the specific heat capacity of air C_p = 1012 J/(kg ˚C), and the plume constant C = 0.17 [7]. Figure 7 is plotted using the model, and shows how changing the rate of air flow, Q_V, impacts the height of the interface between the upper and lower layers, h, (a) and interior temperatures, θ_U and θ_L (b) in the two heating conditions. The comfort criteria are shown as the shaded areas. The plots suggest that when there are 8000 W of distributed heating and 7500 W of localised heating, a flow rate between approximately 2.8 – 2.9 times the minimum ventilation rate, or 2.3 – 2.4 m^3/s, is required to achieve comfort. As the heating rates change to 10000 W of distributed heating and 5500 W of localised heating, the flow rate should be adjusted to between about 2.8 – 3.6 times the minimum ventilation rate, or 2.3 – 3 m^3/s, to maintain comfort. If we assume that pressure loss associated with the inlets and extracts are $c_s = c_e$ = 0.5, then Eqs 2 and 3 suggest that, to achieve the above required flow rates, the areas of the inlets and extracts should be maintained at about $a_s = a_e$ = 4 – 4.3 m^2 in the first case and at about $a_s = a_e$ = 3.8 – 5.8 m^2 in the second case. Such estimation can help inform the design of the envelope of the auditorium at an early stage, and the control of its ventilation system during its operation.

5. Conclusions

We have investigated air flows in a room containing a combination of distributed heating and localised heating, into which cool air is supplied at low momentum through openings at a low level, and from which old air is extracted through openings at a high level. We have addressed a key design and control question, namely: How can we control the internal temperature structure to achieve thermal comfort while maintaining satisfactory ventilation? Using a combination of a theoretical model and laboratory experiments, we have explored the impacts of heating distribution and the rate of air flow on the temperature structure. If the localised heating is sufficiently strong compared to the distributed heating, the room will become stratified into two layers at steady state. A warm layer lies atop a cooler layer which is warmer than the exterior. This temperature structure is controlled primarily by the rate of air flow through the space and the ratio of distributed heating to the total heat flux. For a room with fixed heating, increasing the air flow rate will raise the interface between the upper and lower layers and cool the room. The lower layer is heated primarily by the distributed source, but the upper layer is heated by a combination of the distributed and localised sources. For a fixed air flow rate and a fixed total heat flux, increasing the strength of the distributed source will warm the lower layer but does not change the temperature in the upper layer. The interface between the lower and upper layers also rises as the strength of the distributed source increases. To achieve sufficient ventilation and thermal comfort in the occupied lower zone while keeping it clear of any pocket of uncomfortably warm air, an appropriate range of flow rates is required, which depends on the ratio of distributed heating to the total heat flux. In a naturally operated system in which air supply and extraction are accomplished by buoyancy forces created by the internal heat sources, such appropriate flow rates may be achieved by adjusting the sizes of the inlets and extracts. We have shown how the model may be used to size the areas of the inlets and extracts appropriately for different heating conditions. The results can be used as a basis for developing effective design and control strategies. However, it should be noted that the model presented herein has been developed with an aim to acquire fundamental principles for design and control of a new class of flows, and therefore ignored certain complexities which may be present in certain real-life situations, such as thermal mass or multiple sources of localised heating. To address such complex situations, the present model may be modified or coupled with other appropriate models.

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