Roughness-sublayer correction for the profiles of mean velocity and turbulence over urban areas

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Background
- Monin-Obukhov similarity theory (MOST) applies in inertial sub-layer (ISL) but fails in roughness sub-layer (RSL) because the flow structure in RSL is highly inhomogeneous.
- Extrapolation of the conventional logarithmic law of wall into the RSL likely overlooks the inhomogeneity.
- Need for an analytical expression for mean velocity profile and ventilation estimate, including a new RSL correction, that is applicable over the urban boundary layer.

Analytical Expression for RSL flow correction

Assumptions:
- \( \Phi_m = \Phi_r \phi_m = \phi_r \left( \frac{z}{z_0} \right) \) is a generalised similarity function of ISL & RSL
- Flows above urban canopy in neutral stratification (\( \Phi_m = 1 \))
- \( \Phi_m \) is a function of the roughness elements that is independent from the MOST length scale \( L \)

\[ \Phi_m = \frac{z}{z_0} \left( \frac{z}{z_0} \right)^{- \alpha} \]

The gradient of the wind profile in dimensionless form is,

\[ \frac{du}{dz} = u^* \left( \frac{z}{z_0} \right)^{- \alpha} \]

\( u^* \) is the friction velocity & \( \kappa (= 0.41) \) the von Kármán constant.

Rearrange & integrate yields,

\[ \ln \left( \frac{z}{z_0} \right) = \ln \left( \frac{z}{z_0} \right) - \int_{z_0}^{z} \frac{1 - \frac{1}{\kappa} \phi_m(\frac{z}{z_0})}{\phi_m(\frac{z}{z_0})} \]

\( d \) is the displacement height & \( z_0 \) the roughness length scale.

We employ the (continuous) function of \( \phi_m \)

\[ \phi_m(z) = 1 - e^{-\mu(\frac{z}{z_0})} \]

\( \mu \) is an empirical constant.

Use series expansion to calculate the exponential integral, an analytical expression for the urban RSL effects is formulated

\[ \Phi_m(z) = \frac{1}{\kappa} \left[ \ln \left( \frac{z}{z_0} \right) - \ln \left( \frac{z - d}{z_0} \right) - \sum_{n=1}^{\infty} \frac{(-1)^n \left( \mu \frac{z - d}{z_0} \right)^n}{n!} \right] \]

\( \gamma = 0.57721 \): Euler constant.

Wind Tunnel Measurements
- The open-circuit type wind tunnel at the Department of ME, HKU was used with neutral stratification and a reference wind speed of 9 m s\(^{-1}\)
- Idealised 2D-roughness elements with different aspect ratio (AR = h/b) were used to simulate the urban areas
- Cross-wire hot-wire measurements were performed

Flows and Ventilation Estimates over Idealised Urban Areas
- Flow inhomogeneity over idealised urban areas is revealed (Fig. 3a)
- RSL & ISL are clearly identified
- The newly proposed analytical expression performs well in both RSL & ISL for the prediction of velocity profiles over a wide range of aspect ratios, 0.5 < ARs < 0.083 (Fig. 3c)
- Friction factor \( f \) & vertical velocity scale \( \bar{w} \) are used to parameterise ventilation performance over urban areas with RSL corrections (Fig. 3b)

Next steps
- Tests with additional roughness elements of different forms using wind tunnel experiments, i.e. cube roughness, building height variability or realistic city models.
- Quantify the effect of aerodynamic roughness on RSL flows.
- Examine the RSL turbulence using mixing length models.

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